To appear in Monthly Notices of the R.A.S.

Mon. Not. R. Astron. Soc. **000**, 000–000 (1994) Printed 1 February 2008 (MN plain TEX macros v1.6)

Testing anthropic predictions for Λ and the CMB temperature

J.A. Peacock

Institute for Astronomy, University of Edinburgh, Royal Observatory, Blackford Hill, Edinburgh EH9 3HJ

ABSTRACT

It has been claimed that the observed magnitude of the vacuum energy density is consistent with the distribution predicted in anthropic models, in which an ensemble of universes is assumed. This calculation is revisited, without making the assumption that the CMB temperature is known, and considering in detail the possibility of a recollapsing universe. New accurate approximations for the growth of perturbations and the mass function of dark haloes are presented. Structure forms readily in the recollapsing phase of a model with negative Λ, so collapse fraction alone cannot forbid Λ from being large and negative. A negative Λ is disfavoured only if we assume that formation of observers can be neglected once the recollapsing universe has heated to $T \gtrsim 8$ K. For the case of positive Λ, however, the current universe does occupy a extremely typical position compared to the predicted distribution on the $\Lambda - T$ plane. Contrasting conclusions can be reached if anthropic arguments are applied to the curvature of the universe, and we discuss the falsifiability of this mode of anthropic reasoning.

Key words: galaxies: clustering

1 INTRODUCTION

There is now almost unanimous agreement that the universe contains a component that strongly resembles Einstein's cosmological constant. This conclusion is cross-buttressed by a variety of lines of evidence (e.g. Spergel et al. 2006), and seems unlikely to change. Physically, this means that the vacuum appears to possess a homogeneous energy density, with an associated negative pressure.

The existence of a non-zero vacuum density raises two problems: (1) the scale problem, and (2) the why-now problem. The first of these concerns the energy scale corresponding to the vacuum density. If we adopt the values $\Omega_v = 0.75$ and $h = 0.73$ for the key cosmological parameters, then

$$\rho_v = 7.51 \times 10^{-27}\,\mathrm{kg\,m^{-3}} = \frac{\hbar}{c}\left(\frac{E_v}{\hbar c}\right)^4, \tag{1}$$

where $E_v = 2.39$ meV is known to a tolerance of about 1%. The vacuum density should receive contributions of this form from the zero-point fluctuations of all quantum fields, and one would expect a net value for E_v of order the scale at which new physics truncates the contributions of high-energy virtual particles: anything from 100 GeV to 10^{19} GeV. The why-now problem further asks why we are observing the universe at almost exactly the unique time when this strangely small vacuum density first comes to dominate the cosmic density.

Taking the second problem first, a question that involves the existence of observers must necessarily have an answer in which observers play a role. Therefore, a solution to the why-now problem requires anthropic reasoning. In defining exactly what this term means, it is convenient to draw a distinction between what might be called one-universe and many-universe anthropic arguments. The first of these should not be controversial, since it states that observers are likely to exist at special times in this universe: complex structures cannot begin to form until temperatures reach $T \ll 1000$ K, and the formation of structure will switch off in the near future when conditions become heavily vacuum dominated and the universe enters a phase of exponential expansion. Because the densities in baryons and dark matter are very roughly comparable, and because the photon-to-baryon ratio is roughly $137^2 m_p/m_e$, this window for structure formation also in practice opens soon after matter-radiation equality (e.g. p97 of Peacock 1999), although there is no suggestion that this is anything other than a genuine coincidence. If we further assume that complex life requires metals, it is no surprise that the universe now has an age comparable to a typical stellar lifetime, and this is almost certainly the explanation for Dirac's large-number coincidence (e.g. Carter 1974).

The single-universe expectation that we should live relatively soon after matter-radiation equality may be all that is required of anthropic reasoning. The quintessence programme aims to find a dynamical origin for the vacuum energy; the hope is that the change in cosmological expansion history at matter-radiation equality may prompt the effective vacuum density generated by some scalar field to change from a sub-dominant contribution at early times, to something that resembles Λ by the present. This would be a satisfying solution to the why-now problem, but it is not clear that the mechanism can be made to work. For a simple scalar field with an arbitrary potential, it seems that an

energy scale of ~ 1 meV needs to be introduced into the potential by hand, in order to prevent the quintessence density from tracking the overall mass density at all times (e.g. Liddle & Scherrer 1999). This simply swaps one unexplained coincidence for another. Greater success is achieved with the more radical option of k-essence, in which the field is given a non-canonical kinetic term. This readily achieves departures from tracking, but no simple model has been exhibited in which the late-time behaviour necessarily approaches something close to a constant-density Λ term (Malquarti, Copeland & Liddle 2003).

The other major problem with the quintessence approach is that the models do not solve the scale problem: the potentials asymptote to zero, even though there is no known symmetry that requires this. This leads to consideration of the more radical many-universe mode of anthropic reasoning. Here, one envisages making many copies of the universe, allowing the value of the vacuum density to vary between different versions. The simplest concrete way of generating this multiverse is via stochastic inflation with an additional scalar field that sets the effective value of Λ (e.g. Garriga & Vilenkin 2000). In this paper we will not need to be specific about the mechanism involved, although it is of course of the greatest interest. But first we must establish whether an ensemble approach makes sense. The logic is the same as in evolutionary biology, where we start with empirical evidence for selection out of a diversity of heredity, and only later move on to the search for the microscopic mechanism of genes and DNA that permits this diversity.

Although most members of the hypothetical ensemble are presumed to have large vacuum densities comparable in magnitude to typical particle-physics scales, rare examples will have much smaller densities. Since large values of the vacuum density will inhibit structure formation, observers will tend to occur in models where the vacuum density falls in a small range about zero – thus potentially solving both the scale and why-now problems. The basic form of this argument as an upper limit on the magnitude of Λ has existed for a number of years: see e.g. Linde (1984; 1987) or Barrow & Tipler (1986). A significant further step was taken by Weinberg (1989), who used the anthropic argument to make the impressively bold prediction that Λ would indeed turn out to be non-zero at about the observed level. Weinberg's reasoning was taken up in more detail by Efstathiou (1995) and by Martel, Shapiro & Weinberg (1998). Efstathiou calculated the expected distribution for Ω_v for a typical observer (a term whose meaning is discussed below); he found a result that peaked around $\Omega_v \simeq 0.9$, in which the observed $\Omega_v = 0.75$ would not be surprising.

This is an impressive result, but it has two points at which further study is merited. Although he gave the general argument for the suppression of large observed values of Λ, Efstathiou then fixed the CMB temperature at its observed value in order to calculate a probability distribution for the observed value of Ω_v. In principle, it is possible that $T = 2.725$ K is not a typical value when all observers are considered, and we want to be sure that we have not brought the why-now problem in again by the back door. Garriga, Livio & Vilenkin (2000) have argued that anthropic selection does indeed provide a general solution to the why-now problem, but it seems useful to make this argument explicit in terms of observables. One aim of this paper is therefore to revisit Efstathiou's calculation to calculate the joint distribution of Λ and T, to see how typical our observed location on this plane may be. We also take the chance to apply more accurate approximations for gravitational collapse of cosmic structures than the usual Press-Schechter (1974) approach, which underestimates the abundance of the most massive objects by a factor 10.

A larger issue is the behaviour for negative Λ, which is examined in some detail. Efstathiou concentrated on models that were in the expanding phase, whereas a universe with negative Λ will eventually cease expanding and collapse to a big crunch. New accurate approximations are given for the growth of density fluctuations, for either sign of Λ. It turns out that structure formation in the collapsing phase is highly efficient, which presents something of a puzzle given the observed positivity of Λ.

Finally, the same anthropic approach can in principle also be applied to the case of cosmological curvature. We indicate how this argument might have been applied to explain an open universe in the period before evidence for Λ emerged. The paper concludes by discussing the testability of anthropic reasoning in the light of these results.

2 ANTHROPIC SELECTION AND GRAVITATIONAL COLLAPSE

2.1 Principles

The first step in dealing with ensembles of universes is to decide what will be allowed to vary. In principle, nothing is guaranteed to be fixed, but it makes sense to start in as restrictive a way as possible. We therefore follow Efstathiou (1995): he assumed that all members of the ensemble are exactly spatially flat, and that the key dimensionless ratios of cosmology are as we observe. These are (1) the photon to baryon number density ratio; (2) the dark matter to baryon density ratio; (3) the horizon-scale amplitude of density fluctuations, $\delta_{\rm H}$. The only variation to be considered is in the value of the effective cosmological constant.

Many authors have allowed a wider set of parameters to vary. For example, Garriga & Vilenkin (2006) consider joint variations in Λ and in $\delta_{\rm H}$. The most radical view is that of the string-theory landscape, in which all of physics is free to vary (Susskind 2003), and aspects of this picture have been explored by e.g. Tegmark et al. (2006) and Graesser & Salem (2006). One approach to this question is an experimental one: if the simplest forms of anthropic variation can be ruled out, this might be taken as evidence in favour of the landscape picture. We therefore consider only the simplest picture here. This amounts in practice to considering the observed universe at some high temperature, so that any vacuum density of practical interest is negligible, and then considering the future evolution of copies of this universe in which the vacuum density is set to different levels.

The most difficult and controversial question with any ensemble of universes is how to set a measure: what is the relative weighting of the different members of the ensemble? In the present context, we need to know the prior probability to assign to a given value of Λ, which will be modified by an 'observer-bias' factor, reflecting the relative difficulty of forming observers as a function of Λ. Various proposals for the measure in the case of eternal inflation have been considered by Aguirre, Gratton & Johnson (2006). In the end, we

are persuaded by the original view of Weinberg (1989), who took the prior on Λ to be uniform over a small range around zero (see also Efstathiou 1995 and Weinberg 2000). If there is nothing special about a value zero, this seems a defensible assumption given that the range of interesting densities is tiny by comparison with particle-physics scales.

These different universes are assumed to receive a weight according to the number of observers that exist in them, so that cosmologists asking questions here and now are treated as randomly selected from the totality of observers over all universes and all times. The exact meaning of this intuitive idea of a 'typical' observer is not so easily made precise, however, since observers are not standard objects (what relative weight do we assign to a modern human, a caveman and a cat?). Some of these difficulties are reviewed comprehensively by Neal (2006). In the present instance, we can side-step many of these issues by exploiting the assumed similarity of the members of the ensemble in their non-vacuum physics. We do not need to predict the absolute number of observers, nor how they are divided into different types of observer: it is sufficient to assume that a model with twice as many stars is twice as likely to be experienced. Thus, we take the weighting of each member of the ensemble to be given by the fraction of the baryons that are incorporated into nonlinear structures.

Weighting by collapse fraction is a common assumption in work of this sort, although not universally accepted. The most recent dissenters are Starkman & Trotta (2006), who calculate the maximum number of observations a given observer can ever make, which introduces a further dependence on Λ. There is certainly an ambiguity worth debating here: given two universes that make the same number of observers, but where one set live much longer than the other, should we give an equal weight to each, or weight them in proportion to lifetime? Starkman & Trotta effectively treat equal time intervals as equally probable, but the instants in the life of a given individual are not independent: any of us will always answer "yes" to the question, "is Λ surprisingly low?", and it should not matter how many times we are able to ask. Therefore, we prefer to weight by the numbers of observers produced, and it seems reasonable to tie this to the number of stars that are formed.

As discussed earlier, we need a prior for the different values of the vacuum density: following Efstathiou (1995) and Weinberg (1989; 2000), this is taken as being uniform over a range around zero. We therefore weight universes with an overall posterior probability

$$dP(\rho_v) \propto f_c \, d\rho_v, \tag{2}$$

where f_c is the collapse fraction: the proportion of mass in the universe that has become incorporated into sufficiently large nonlinear objects.

In more detail, we will be interested in the weight attaching to different times in each universe. This is in part a simple issue, since we can calculate what fraction of the mass undergoes collapse in any given time interval, so that

$$dP(\rho_v, t) \propto \frac{df_c}{dt} \, dt \, d\rho_v. \tag{3}$$

But this gives the time distribution for the *formation* of sites at which life might subsequently emerge. The more serious challenge lies in predicting the history of observers following a formation event: how many observers will eventually result, and what will their distribution in time be? Rather than making arbitrary assumptions, we can avoid the worst uncertainties by turning the problem backwards. We can calculate the distribution of times at which stars form in the universe, and we know when the star with which we are associated was formed: 4.6 Gyr ago. That time corresponds to a redshift 0.457, at which point the cosmological parameters were $T = 3.97$ K and $\Omega_v = 0.49$. We can therefore concentrate on the more concrete question of whether the sun formed at a typical point in comparison to all stars in the multiverse. This presumes that the subsequent history of life around each star has no dependence on Λ, but further biological assumptions are not required.

2.2 The collapse fraction

For this calculation, we need to be able to predict the fraction of baryons in the universe that are processed into stars. This is not something that can presently be calculated without some guidance from observation. The first galaxy-sized systems to collapse are of low mass and high density, and clearly will form some stars, but it is expected that feedback from this initial activity will quickly regulate the star formation in these objects. Thus in practice most star formation is expected to occur in the largest galaxies, and we follow Efstathiou (1995) in treating these as being defined by a single mass scale. However, as shown below, it is now possible to deduce this scale empirically, rather than appealing to a priori cooling arguments such as Rees & Ostriker (1977).

We develop below the formulae needed to calculate accurately the fraction of the mass that has collapsed into objects of a given mass scale or larger. As usual, the mass scale is defined by the mass in a homogeneous universe contained within a sphere of radius R. The fractional density fluctuations smoothed with such a spherical filter have an rms value $\sigma(R)$, and the rareness of objects of a given mass is quantified by defining

$$\nu \equiv \delta_c / \sigma(R), \tag{4}$$

where δ_c is a density threshold of order unity. Since σ changes with time, we need to specify an era in order to associate ν with a given mass. It is convenient to make the arbitrary choice of $T = 1000$ K as a reference era (matter dominates over radiation and over any vacuum density of interest). Fig. 1 then shows that the existing data on the evolving comoving stellar density can be well described using a single-scale model with

$$\nu(T = 1000) = 250, \tag{5}$$

adopting $\delta_c = 1.686$ as justified below.

This plot does not need to make any assumptions about the matter power spectrum, but if we adopt a canonical model ($\Omega_m = 0.25$, $\Omega_b = 0.04$, $h = 0.73$, $n_s = 0.95$, $\sigma_8 = 0.8$), this corresponds to an effective galaxy mass of

$$M_g = 1.9 \times 10^{12} \, M_\odot. \tag{6}$$

The small-scale CDM power spectrum is rather flat:

$$d \ln \nu / d \ln M \simeq 0.145, \tag{7}$$

so the results of this calculation should be relatively robust. Loeb (2006) is correct to point out that, in principle, some

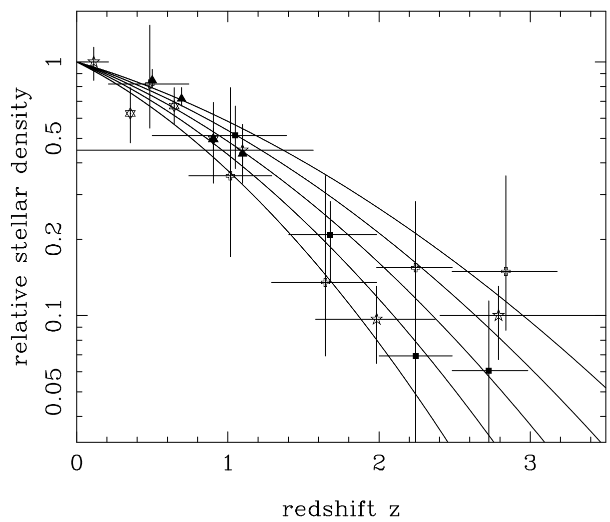

Figure 1. Time dependence of star formation predicted in a simple collapse model. The total stellar density produced by a given epoch is assumed to scale with the total collapse fraction associated with a single mass scale. The density-fluctuation parameter $\nu(T = 1000)$ is varied by up to 20 per cent either side of its canonical value $\nu = 250$. This yields a good match to the data on the empirical redshift dependence of the total stellar mass density, taken from Merloni, Rudnick & Di Matteo (2004).

stars could form in dwarf galaxies, but changing the critical mass scale by a power of ten has little impact on the results.

Even so, it is clear that this simple model can be challenged, since the typical galaxy mass arises in a complex way, where feedback from supernovae and AGN heats gas and prevents star formation being very efficient (the observed cumulative efficiency today is $\Omega_*/\Omega_b \simeq 5\% - 9\%$, depending on assumptions about the IMF: Cole et al. 2001). It is certainly possible that the operation of such effects in objects of a given mass could depend on density and thus on era – so that the overall efficiency of star formation could have a further complicated dependence on Λ. We shall ignore this point here, but it would clearly be of interest to investigate the issue elsewhere using detailed galaxy formation models.

2.3 The cosmological mass function

Given a filtering scale corresponding to a typical galaxy, the linear density contrast, $\sigma(R)$ can be calculated. According to the Press-Schechter (1974) approximation, the collapse fraction (i.e. proportion of mass contained in objects of the given mass scale or larger) is then

$$f_c = \mathrm{erfc}\left(\nu/\sqrt{2}\right); \quad \nu \equiv \delta_c/\sigma. \tag{8}$$

The critical density contrast is $\delta_c \simeq 1.686$ in an Einstein–de Sitter model. Efstathiou (1995) gives an approximate scaling as $(1-\Omega_v)^{-0.28}$, but this is a theoretical expectation based on the spherical collapse model. In detailed studies of numerical simulations, Jenkins et al. (2001) found that δ_c could be treated as constant in matching theory to the simulated mass functions. Jenkins et al. followed Sheth & Tormen (1999), who established that the Press-Schechter form for the mass function was significantly in error, with too many objects at the peak of the mass function by about a factor 1.5, and too few at the highest masses by a power of 10. More recently, Warren et al. (2006) have shown that the Jenkins et al. fitting formula still contains errors of order 10%, and they proposed a replacement. An unsatisfactory feature of their fit is that it predicts collapse fractions in excess of unity for small masses, whereas one would normally prefer to assume that $f_c \to 1$ in the limit of very small masses. It is simple to cure this by finding an analytic formula for f_c; this can then be differentiated in order to find the mass function. The following expression matches the Warren et al. fitting formula to a maximum error of about 1% over the whole range where data exist:

$$f_c = (1 + a\,\nu^b)^{-1} \exp(-c\,\nu^2), \tag{9}$$

where $(a, b, c) = (1.529, 0.704, 0.412)$.

2.4 Growth of density perturbations

For this exercise, we also require the linear growth function for density perturbations, which can be expressed as a numerical integral (Heath 1977). It is convenient to have an accurate numerical approximation, and the following expressions are good to a maximum error of 0.1%. The cases of positive and negative Λ are somewhat distinct. For the positive case,

$$\delta(a) \simeq x(1 - x^{1.91})^{0.82} + 1.437\left(1 - (1 - x^3)^{2/3}\right), \tag{10}$$

where x denotes $\Omega_v(a)^{1/3}$, and we choose the $a = 1$ point to correspond to equal density in matter and vacuum:

$$\Omega_v(a) = (1 + a^{-3})^{-1}, \tag{11}$$

so that $\delta(a) \simeq a$ for small a. For a starting point where $\Omega_v(a)$ is small, the total amount of growth is

$$\delta_\infty/\delta_{\rm init} \simeq 1.437/a_{\rm init} \simeq 1.437/[\Omega_v(a_{\rm init})]^{1/3}. \tag{12}$$

Since $\Omega_v(a_{\rm init}) = \rho_v/5.375 \times 10^8\,\mathrm{meV}^4$ for our choice of $T = 1000$ K as a normalization point, at which the galaxy-scale fluctuation is $\nu = 250$, this immediately allows the asymptotic value of ν to be deduced.

For the negative density case, we need time as a coordinate, since the scale factor is not monotonic:

$$a(t) = [\sin(3t/2)]^{2/3}, \tag{13}$$

where here we choose units such that $a = 1$ at the point of maximum expansion, and time is measured in units of $(8\pi G|\rho_v|/3)^{-1/2}$, so that Friedmann's equation is $(\dot a/a)^2 = a^{-3} - 1$ and $\Omega_v(a) = (1 - a^{-3})^{-1}$. Here, the approximation for the growth function is

$$\delta(t) = \frac{(3t/2)^{2/3}}{(1 + 0.37(t/t_{\rm coll})^{2.18})\,(1 - (t/t_{\rm coll})^2)}. \tag{14}$$

Again, the normalization is that $\delta(a) \simeq a$ for small a. Note that the fluctuations diverge at the collapse time ($t_{\rm coll} = 2\pi/3$) as $1/(t_{\rm coll} - t)$: this corresponds to the decaying mode in the expanding phase.

3 THE EXPECTED SIGN OF Λ

The collapsing phase turns out to be important when we consider structure formation. A substantial negative Λ limits the amount of growth that can occur before the universe

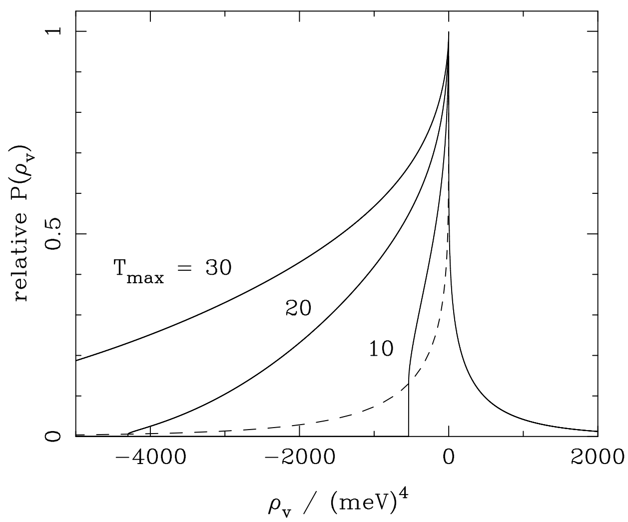

Figure 2. The collapse fraction as a function of the vacuum density, which is assumed to give the relative weighting of different models. The dashed line for negative density corresponds to the expanding phase only, whereas the solid lines for negative density include the recollapse phase, up to maximum temperatures of 10 K, 20 K, 30 K. The observed value of the vacuum density is 33 meV^4.

ceases to expand, but the total amount of growth after this is limited only by how close to the big crunch we are prepared to venture. Eventually, f_c tends to unity in all such recollapsing models. Since the uniform prior extends arbitrarily far towards more negative values of Λ, this apparently implies that all the weight should be given to models with $\Lambda < 0$.

However, structures that form very close to the final singularity are not of interest for the anthropic calculation: there is little time remaining for life to develop, and in any case the CMB will have heated up to the point where it interferes with life – or indeed perhaps even with the formation of stars and planets themselves. It is simplest to express this cutoff in the recollapsing phase in terms of a maximum temperature that we are willing to consider, although this can be directly translated to a limit on the time remaining before the big crunch. Since the recollapsing phase is the time-reversed version of the expansion, the time remaining from temperature T until the big crunch is just what would have elapsed from the big bang until this temperature. Normally, the matter-dominated approximation will apply, so

$$t(T) \simeq \frac{2}{3H_0}\,\Omega_m^{-1/2}\,(1+z)^{-3/2} = \left(\frac{T}{18.6\ \mathrm{K}}\right)^{-3/2}\ \mathrm{Gyr}. \qquad (15)$$

We know from observations that star formation in galaxies can proceed actively at redshift $z \simeq 7$, so $T_{\max} > 10$ K on these grounds. This would leave only a few Gyr after formation for life to evolve, so presumably $T_{\max}$ should not be much larger than this, and could well be smaller. This is not so much a biological argument as one based on stellar lifetimes. Highly negative values of Λ would cause the universe to turn round without ever cooling below this critical temperature, so these models may be excluded from the point of view of generation of observers, even though they form galaxy-scale structures efficiently. This is completely distinct from the situation at highly positive Λ, where the problem is the failure to create structure at any time.

Rather than singling out a particular value of $T_{\max}$, we may as well perform the calculation allowing it to take a range of values. This is illustrated in Fig. 2, which shows the relative weight to be given to models as a function of ρ_v. We can integrate this distribution to obtain Fig. 3, which shows how the probability of observing a negative Λ varies with the maximum temperature for structure formation that we are willing to tolerate. The probability that observers inhabit a universe with $\Lambda < 0$ is about 50% if $T_{\max} = 8.5$ K, or as little as 1% if $T_{\max} = 1.5$ K. Conversely, only about 6% of observers will experience a *positive* Λ if $T_{\max} = 30$ K; but this is probably too tolerant of late-forming observers. From Fig. 2, it is furthermore clear that the bulk of any weight in favour of negative Λ lies with the recollapsing phase: the probability of inhabiting the expanding phase is approximately the same as the probability that Λ is positive. According to the anthropic framework, it is therefore far from inevitable that we have ended up in a positive-vacuum expanding universe, although neither is it particularly unusual.

We can now make a first attempt to assess how well this anthropic prediction matches reality. Since $\Lambda = 0$ is a special point, it is reasonable to consider the frequentist question: 'what is the probability that $|\rho_v|$ lies within $(2.39\,\mathrm{meV})^4$ of the origin?'. The answer is also plotted in Fig. 3: about 10% if we ignore negative Λ, peaking at 20% for $T_{\max} = 4$ K, and declining for larger values. The consistency of observations with anthropic prediction therefore depends somewhat on the recollapsing phase. In what follows, we will concentrate on the expanding case of positive Λ, but it should be borne in mind that any anthropic probabilities we deduce subsequently should be reduced slightly to account for the fact that observers also have a non-negligible chance of being found in a recollapsing model.

4 THE JOINT DISTRIBUTION OF Λ AND T

The above calculation addresses the scale problem, but only considers the total amount of structure formation, not when it occurs. This problem was considered by Garriga, Livio & Vilenkin (2000), who calculated the posterior distribution on the plane $(t_\Lambda, t_{\rm G})$, where t_Λ is the time of Λ-domination and $t_{\rm G}$ the time at which a typical galaxy was formed. They used a Press-Schechter approach to show that the probability density on this plane peaks around $t_{\rm G} \sim t_\Lambda$, so that the why-now problem is solved. We need something similar, but we want to see explicitly how the Efstathiou analysis is affected by a given assumed temperature; it is therefore necessary to cast the distribution in terms of the observables Ω_v and T. We have the differential probability distribution $dP \propto d\rho_v\, df_c$, and we want to change variables to Ω_v and T. Rather than doing this in a single step by working out the Jacobian of the transformation, we can first note that the collapse fraction is just a function of T for given ρ_v, so that

$$dP \propto d\rho_v\ \left.\frac{\partial f_c}{\partial T}\right|_{\rho_v} dT. \qquad (16)$$

If we now change from (ρ_v,T) to (Ω_v,T), the Jacobian is diagonal, and

$$dP \propto \left.\frac{\partial \rho_v}{\partial \Omega_v}\right|_T\ \left.\frac{\partial f_c}{\partial T}\right|_{\rho_v}\ d\Omega_v\ dT. \qquad (17)$$

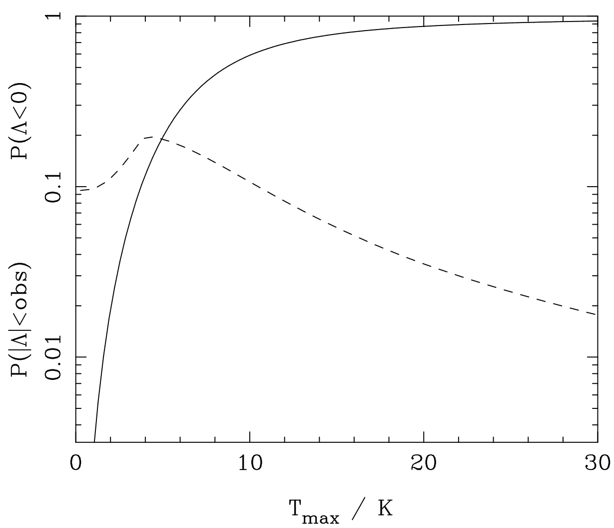

Figure 3. Integrating under the distribution of Fig. 2, we can deduce the probability of inhabiting a universe with $\Lambda < 0$, as a function of the assumed maximum temperature for galaxy formation (solid line). The dashed line shows, under the same assumption, the probability that the absolute value of Λ would lie within its observed value of $(2.39\,\mathrm{meV})^4$. For a maximum temperature of 8.5 K, positive and negative values of Λ are equally probable, but negative Λ is disfavoured by more stringent limits on temperature: for a maximum temperature of 1.5 K, only about 1% of observers will inhabit a universe with $\Lambda < 0$.

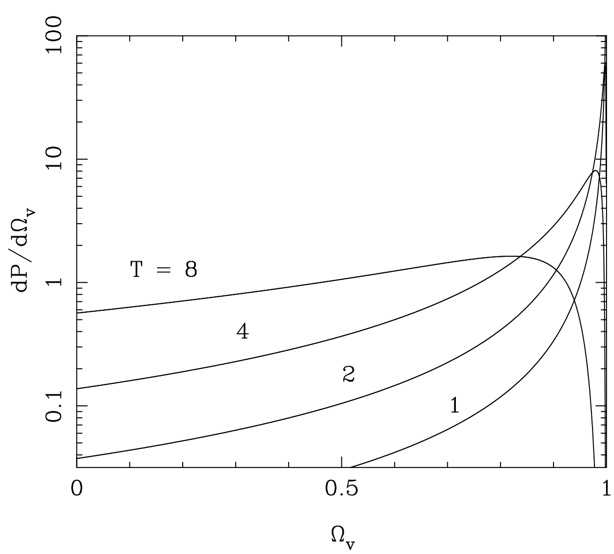

Figure 4. The probability distribution of Ω_v at observed temperatures of 1 K, 2 K, 4 K, 8 K, with higher temperatures pushing the distribution to lower values of Ω_v. This plot should be contrasted with Fig. 2 of Efstathiou (1995).

To evaluate the first factor on the rhs, note that the vacuum density parameter at some early time t_0 is

$$\Omega_{v0} = \frac{\Omega_v}{\Omega_v + (1-\Omega_v)a_0^{-3}} \simeq \frac{\Omega_v}{(1-\Omega_v)a_0^{-3}}, \tag{18}$$

where Ω_v is the density parameter at the later time of interest, and $a_0 = T/T_0 \to \infty$ gives the latter approximation. In this limit, $\Omega_{v0} \ll 1$; this is therefore proportional to ρ_v, giving

$$\left.\frac{\partial \rho_v}{\partial \Omega_v}\right|_T \propto \frac{T^3}{(1-\Omega_v)^2}. \tag{19}$$

This gives higher weight to Ω_v close to 1, since this is an attractor for the evolution of $\Omega_v(t)$ if $\Lambda > 0$. The focusing towards $\Omega_v = 1$ increases as T falls, and this is reflected in the T^3 factor. As a result, dP/dT with Ω_v fixed at zero differs in this framework from what we would normally calculate for an Einstein–de Sitter universe.

Efstathiou (1995) did not need to consider the T^3 factor, since he held the temperature constant at its observed value. This is appropriate if we want to use the anthropic framework in a Bayesian sense, to make the best *prediction* of the current value of Ω_v given what else we know. For similar reasons, Efstathiou held constant the observed large-angle CMB fluctuations, whereas these vary with temperature (i.e. with time of observation). Here, we are interested in the broader question of whether the conditions we observe are close to those experienced by a typical observer. To show that this distinction matters, consider Fig. 4. This plots the posterior probability of Ω_v for various choices of the observed temperature, and shows that the result is sensitive to temperature. For $T \simeq 8$ K, the distribution peaks near the observed $\Omega_v = 0.75$, but for lower T the distribution is dominated by the spike in the prior at $\Omega_v = 1$.

We now return to the full joint distribution for Ω_v and T, which we had in the form

$$dP \propto \frac{T^3}{(1-\Omega_v)^2}\,\left.\frac{\partial f_c}{\partial T}\right|_{\rho_v}\, d\Omega_v\; dT. \tag{20}$$

The remaining partial derivative is

$$\left.\frac{\partial f_c}{\partial T}\right|_{\rho_v} = \frac{\partial f_c}{\partial \ln \nu}\, T^{-1}\, \frac{\partial \ln \nu}{\partial \ln T}. \tag{21}$$

For convenience, we can use the Peebles (1980) approximation for the logarithmic growth rate:

$$\frac{\partial \ln \nu}{\partial \ln T} \simeq (1-\Omega_v)^{0.6}, \tag{22}$$

so that overall

$$dP \propto \frac{\partial f_c}{\partial \ln \nu}\, T^2\, (1-\Omega_v)^{-1.4}\, d\Omega_v\; dT. \tag{23}$$

This joint probability distribution is show in Fig. 5, converting to $\log T$ for convenience. The 'observed' universe of $(T, \Omega_v) = (3.97, 0.49)$ is plotted as a point. It is clear that the point does not lie in a particularly unusual position in this plane. If we draw contours of constant probability density, the point lies well within the 68% contour. These contours are not unambiguous, as they depend on the measure adopted on the $\Omega_v - T$ plane. However, if we inspect the marginalized distributions for Ω_v and T, shown in Figs 6 & 7, we see that the observed conditions are close to the 50% point in each quantity. In short, the anthropic calculation suggests that we are indeed extremely typical observers, both in terms of the vacuum density we see, and when we see it.

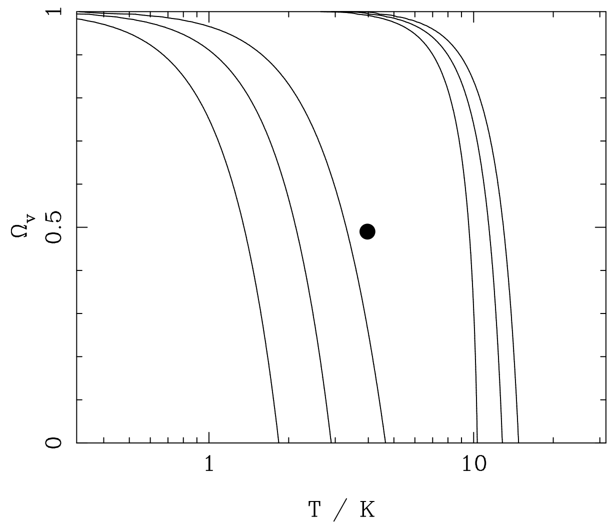

Figure 5. Contours of probability density on the $\log(T) - \Omega_v$ plane. The three contours shown enclose 68%, 95% and 99% of the probability. The solid point shows the conditions at the epoch of formation of the sun.

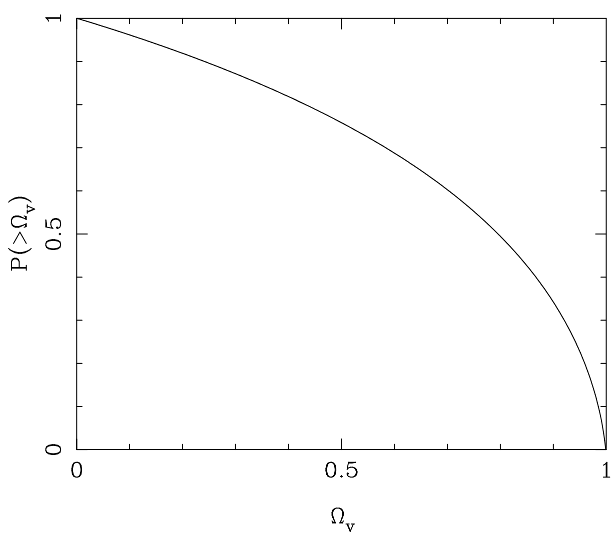

Figure 6. The integral probability distribution for Ω_v, marginalized over T.

5 ANTHROPIC ARGUMENTS APPLIED TO CURVATURE

So far, we have assumed that the vacuum is indistinguishable from a cosmological constant, with equation of state $w = P/\rho c^2 = -1$. We know that this is a good approximation for our universe, and it would in any case go beyond the scope of this paper to consider ensembles in which more than one parameter varies. However, it is worth paying some attention to the special case $w = -1/3$. With the critical exception of the distance-redshift relation, a flat model with this vacuum equation of state is indistinguishable from a model with non-zero spatial curvature: the Friedmann equation and the growth equation for density perturbations are identical. We can then use a modified version of the above

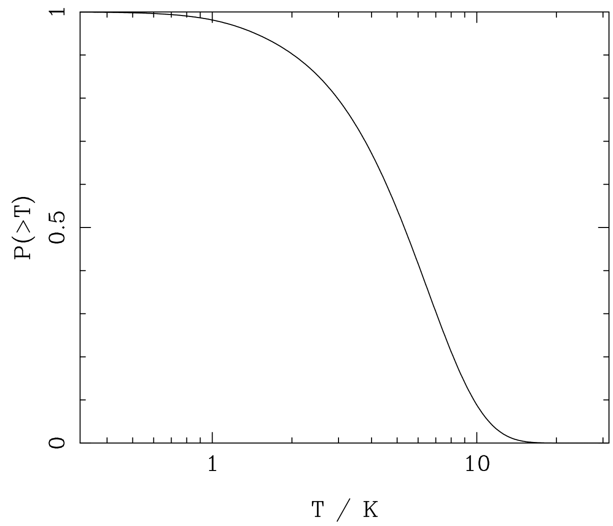

Figure 7. The integral probability distribution for T, marginalized over Ω_v.

approach to show what happens if we confront anthropic reasoning with the curvature of the universe.

To some extent, curvature presents a parallel set of problems to the vacuum. There is a scale problem, in the sense that natural initial conditions might be thought to have a total $|\Omega - 1|$ of order unity, which would lead to a universe dominated by curvature long before today. There could also be a why-now problem, if the present curvature was non-zero at the level of $|\Omega - 1| \sim 0.01$, which cannot currently be excluded. It is commonly assumed that inflation solves the curvature scale problem and also predicts that there is no why-now problem, but it is interesting to see how the anthropic apparatus copes with this case. This issue has been examined previously (e.g. by Garriga, Tanaka & Vilenkin 1999), but it is of some interest to see how our specific approach works out in this case. Rather than break our rule of allowing only one parameter to vary in the ensemble, we take a historical approach and imagine how anthropic arguments might have been applied to curvature decades ago, when many cosmologists were convinced that $\Lambda = 0$ (it is perhaps surprising that anthropic ideas received little emphasis at this time).

Unlike the vacuum density, curvature lacks an obvious time-independent absolute scale. At any given era, one can define $\Omega_k \equiv 1 - \Omega_m$, so it will be convenient to consider $\Omega_k(T = 1000)$ as our parameter. This evolves as $\Omega_k(a) = \Omega_k/(\Omega_k + \Omega_m/a)$. Interesting values of this number at the reference $T = 1000\,\mathrm{K}$ will be small, so it is tempting to follow our previous procedure and assign a uniform prior around zero, and weight models by their asymptotic collapse factor. In the days before inflation, this might have been a defensible expression of ignorance: the essence of the flatness problem is that order unity positive or negative curvature in the initial conditions seems more natural than the tiny amount necessary to yield an almost flat universe today. But in modern models where 'pocket' universes are formed by tunnelling, the result is an open universe, so priors on curvature might well have a discontinuity at zero (e.g. Freivogel et al. 2006). The idea of a uniform prior for curvature is therefore less well founded than it is for Λ. Nevertheless,

it is of some interest to carry out the exercise of adopting a uniform prior and seeing where it leads.

For models with $\Lambda = 0$, the perturbation growth as a function of a is analytic:

$$\begin{aligned}\delta(a) &= +1 + \frac{3}{a^{3/2}}\left(\sqrt{a} - \sqrt{1+a}\,\sinh^{-1}\sqrt{a}\right) \quad (\Omega_k > 0)\\ &= -1 + \frac{3}{a^{3/2}}\left(\sqrt{a} - \sqrt{1-a}\,\sin^{-1}\sqrt{a}\right) \quad (\Omega_k < 0);\end{aligned} \tag{24}$$

in the collapsing phase for negative Ω_k, $\sin^{-1}\sqrt{a}$ is replaced by $\sin^{-1}\sqrt{a} - \pi$. These expressions are normalized so that $\delta \simeq 2a/5$ for $a \ll 1$. The convention for the scale factor assumes that a can be written in terms of conformal time, η as

$$\begin{aligned}a &= (1 - \cos\eta)/2 \quad \text{(closed)}\\ a &= (\cosh\eta - 1)/2 \quad \text{(open)},\end{aligned} \tag{25}$$

so that $a = 1$ at maximum expansion in the closed model. The relation to density parameters in these units is

$$\begin{aligned}\Omega_k(a) &= a/(a-1) \quad \text{(closed)}\\ \Omega_k(a) &= a/(a+1) \quad \text{(open)}.\end{aligned} \tag{26}$$

We can now repeat the exercise of Fig. 2 for the case of curved universes with $\Lambda = 0$, and the results are shown in Fig. 8. Generally, the anthropic weighting as a function of curvature looks similar to the weighting as a function of Λ, but with some important differences. The asymmetry in favour of recollapsing models is not so extreme: the probability of experiencing $\Omega_k > 0$ (i.e. a negatively curved open universe) is 41% for $T_{\rm max} = 10$ K, falling to 29% for $T_{\rm max} = 30$ K.

What about the magnitude of curvature? A decade ago, open models were seriously under consideration, and some would have argued for $\Omega_k \simeq 0.7$, so that $\Omega_k(T = 1000) \simeq 0.006$. From Fig. 8, we see that the typical curvature predicted for such Λ-free universes was $\Omega_k(T = 1000) \simeq 0.01$, so an anthropic approach to explaining the density parameter in matter-only models would have yielded sensible answers. In order to reject an anthropic explanation for an open universe at the 1% level, it would have been necessary to have a limit of $|\Omega_k(T = 1000)| \lesssim 10^{-4}$ (depending slightly on limiting temperature), corresponding to $|\Omega_k| \lesssim 0.035$ at $T = 2.725$. In fact, we barely know that the universe is flat to this precision even today: the present limit is approximately $|\Omega_k| < 0.02$, according to Spergel at al. (2006). Therefore, an anthropic approach to curvature would have been perfectly consistent with 1990s data.

Today, the issue of curvature would be approached in the context of inflation, where a sufficient number of e-foldings, N, of the expansion will lead to values of the present curvature that are unmeasurably small. The issue of interest is therefore the prior to be placed on N (see e.g. Freivogel et al. 2006). Whatever the result of such a calculation, however, a sufficiently small upper limit on curvature would allow the anthropic argument to be rejected. Similarly, if we had no detection of Λ, our earlier results show that a sufficiently strong upper limit would reject the anthropic approach, leading us to require a physical mechanism that forces $\Lambda = 0$. Anthropic reasoning is thus testable and could point to new physics. But this is not the situation we face: we have an actual detection of Λ, rather than an ever-retreating upper limit, and no a priori theory predicts the observed number. An explanation in terms of anthropic selection from an ensemble matches what we see, and so far there is no credible alternative.

6 CONCLUSIONS

This paper has revisited the anthropic calculation of Efstathiou (1995) in more detail, dropping the assumption that the CMB temperature is fixed, and considering also the formation of structure during the collapsing phase of a model with negative vacuum density. We adopt Weinberg's (1989) assumption that the prior on Λ is uniform in a small range around zero, so that models are weighted simply by their collapse function. We have attempted to avoid issues such as the lifespan of civilizations by using the formation era of the sun as the data to explain: so long as observers eventually form with some fixed mean number per star, we can ignore how they are subsequently distributed in time. For universes that are expanding, the conclusion is that the sun formed at completely typical values of Ω_v and the CMB temperature. There is therefore no observational reason to challenge the idea of a uniform prior on Λ, nor to require variation of other parameters within the ensemble – although there remains theoretical motivation for considering more complex alternatives.

This approach can certainly be questioned in the case of negative vacuum densities, where the age of the universe is finite. One might be tempted to argue that all the weight should go to positive-density universes, since civilizations can then potentially last forever. However, this is too optimistic, since the event horizon in models that are asymptotically de Sitter limits the resources that are available. Once most of the stars in a given model have formed, it is reasonable to expect that observers will find existence progressively more difficult after a further few billion years as the existing stars die out. In any case, we know that we are members of a civilization that has not yet outlived its star; the question of whether some races might live for a trillion years can therefore be ignored. The reasoning here is the same as in the "God's coin toss" thought experiment discussed by Olum (2002). This experiment imagines that according to the toss of a coin either 10 (heads) or 1000 (tails) people are created and given a number. If you have no knowledge of your number, your odds for the result of the coin toss should be 100:1 in favour of tails; but if your number is ≤ 10, your odds should be equal.

From this point of view, there thus seems no reason not to give appropriate weight to the flourishing of observers in recollapsing models. In contrast to models with positive vacuum density, gravitational collapse is always perfectly efficient in models where $\Lambda < 0$. But this does not imply that anthropic selection must favour this case: temperatures should not too be too high at putative formation, and there should remain at least a few billion years before further collapse renders the CMB hot enough to be an environmental hazard. These criteria motivate a maximum temperature of order 10 K, for which only a minority ($\sim 10\%$) of observers should find themselves in recollapsing models. This was not our fate; but it is interesting to speculate how observational cosmology might have developed in such a case.

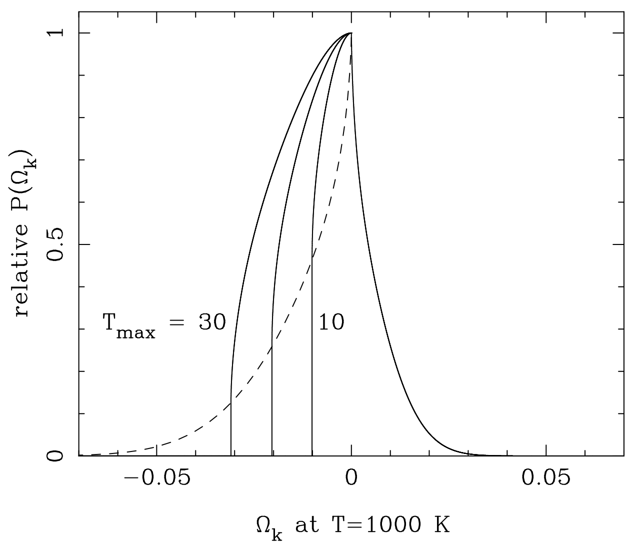

Figure 8. The collapse fraction as a function of the curvature in models with $\Lambda = 0$, which is assumed to give the relative anthropic weighting of different models. Curvature is specified as $\Omega_k = 1 - \Omega_m$ at the era when $T = 1000$ K. The dashed line for negative Ω_k corresponds to the expanding phase only, whereas the solid lines for negative Ω_k include the recollapse phase, up to maximum temperatures of 10 K, 20 K, 30 K.

ACKNOWLEDGEMENTS

This project was carried out under the support of a PPARC Senior Research Fellowship. Thanks are due to Bernard Carr, Andrei Linde, Roberto Trotta Alex Vilenkin and particularly George Efstathiou for comments in the course of this work.

REFERENCES

Aguirre A., Gratton S., Johnson M.C., 2006, hep-th/0611221
Barrow J.D., Tipler F.J., 1986, *The Anthropic Cosmological Principle*, Oxford University Press
Carter B., 1974, Proc. IAU Symposium no. 63 (Reidel), p291
Cole S. et al. (the 2dFGRS Team), 2001, MNRAS, 326, 255
Efstathiou G., 1995, MNRAS, 274, L73
Freivogel B., Kleban M., Rodriguez Martinez M., Susskind L., 2006, JHEP 0603, 039 (hep-th/0505232)
Garriga J., Tanaka T., Vilenkin A., 1999, Phys. Rev. D, 60, 023501
Garriga J., Livio M., Vilenkin A., 2000, Phys. Rev. D, 61, 023503
Garriga J., Vilenkin A., 2000, Phys. Rev. D, 61, 083502
Garriga J., Vilenkin A., 2006, Prog. Theor. Phys. Suppl. 163, 245 (hep-th/0508005)
Graesser M.L., Salem M.P., 2006, astro-ph/0611694
Heath D., 1977, MNRAS, 179, 351
Jenkins A.R., Frenk C.S., White S.D.M., Colberg J.M. Cole S., Evrard A.E., Couchman H.M.P., Yoshida N., 2001, MNRAS, 321, 372
Liddle A.R., Scherrer R.J., 1999, Phys. Rev. D 59, 023509
Linde A.D., 1984, Rept. Prog. Phys., 47, 925
Linde A.D., 1987, in *300 Years of Gravitation*, ed. S.W. Hawking and W. Israel, Cambridge University Press, p604
Loeb A., 2006, JCAP, 0605, 009 (astro-ph/0604242)
Malquarti M., Copeland E.J., Liddle A.R., 2003, Phys. Rev. D, 68, 023512
Martel H., Shapiro P., Weinberg S., 1998, ApJ, 492, 29
Merloni A., Rudnick G., De Matteo T., 2004, MNRAS, 354, L37
Neal R.M., 2006, math.ST/0608592
Olum K.D., 2002, Phil. Q., 52, 164 (gr-qc/0009081)
Peacock J.A., 1999, *Cosmological Physics* (Cambridge)
Peebles P.J.E., 1980, *The large-scale structure of the universe* (Princeton)
Press W.H., Schechter P., 1974, ApJ, 187, 425
Rees M.J., Ostriker J.P., 1977, MNRAS, 179, 541
Sheth R.K., Tormen G., 1999, MNRAS, 308, 119
Spergel D.N. et al., 2006, astro-ph/0603449
Starkman G.D., Trotta R., 2006, astro-ph/0607227
Susskind L., 2003, hep-th/0302219
Tegmark M., Aguirre A., Rees M.J., Wilczek F., 2006, Phys.Rev. D73, 023505 (astro-ph/0511774)
Warren M.S., Abazajian K., Holz D.E., Teodoro L, 2006, ApJ, 646, 881 (astro-ph/0506395)
Weinberg S., 1989, Rev. Mod. Phys. 61, 1
Weinberg S., 2000, Phys. Rev. D, 61, 103505

This paper has been produced using the Royal Astronomical Society/Blackwell Science TEX macros.